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ABSTRACT

Course placement systems in the first year of college include both an assessment component (to estimate students' probability of success in standard courses) and an instructional component (in which underprepared students are taught the skills and knowledge they need to succeed in the standard courses). The effectiveness of a placement system depends on students' eventual success in the standard courses, usually defined in terms of completing these courses with satisfactory grades. This paper illustrates how statistical theory can be used to model the effectiveness of aspects of a course placement system. The illustration is based on data from 268 students who enrolled in different first-year mathematics courses at a midwestern university. First, students' preferences for different course grades were estimated separately under the assumption that they had or had not taken remedial courses. Students expressed their preferences in terms of "value functions." The actual outcomes for students who received remedial instruction before enrolling in a course were then compared with their predicted outcomes had they not done so. An analogous comparison is also made between students' expected value assumptions under the two assumptions. Remedial instruction increased students' probability of success in standard courses. Its principal effect was to decrease the proportion of students who received "W"s and "F"s slightly, and to increase the proportion of students who received "D"s, "C"s, and "B"s. This benefit was not large enough, however, to offset the "disutility" associated with taking remedial courses as expressed by students' value functions. Appendixes discuss statistical issues and logistic regression results and present the value function questionnaire. (Contains 8 tables, 10 figures, and 39 references.) (Author/SLD)

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Indicators of the Effectiveness of Developmental Course Placement Systems in College

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Abstract

Course placement systems in the first year of college include both an assessment component (to estimate students' probability of success in standard courses), and an instructional component (in which underprepared students are taught the skills and knowledge they need to succeed in the standard courses). The effectiveness of a placement system depends on students' eventual success in the standard courses. Success is usually defined in terms of completing these courses with satisfactory grades.

This paper illustrates how statistical decision theory can be used to model the effectiveness of aspects of a course placement system. The illustration is based on data from students who enrolled in different first-year mathematics courses at a midwestern university. I first estimated students' preferences for different course grades, separately under the assumption that they either had or had not previously taken remedial courses. The students expressed their preferences in terms of 'value functions'. I then compared the actual outcomes of students who received remedial instruction before enrolling in the courses with their predicted outcomes, had they not done so. I also made an analogous comparison between students' expected value functions under the two assumptions.

Remedial instruction increased students' probability of success in standard courses. Its principal effect was to decrease slightly the proportion of students who received Ws and Fs, and to increase the proportion of students who received Ds, Cs, and Bs. This benefit was not large enough, however, to offset the disutility associated with taking remedial courses, as expressed by the students' value functions.

Indicators of the Effectiveness of Developmental Course Placement Systems in College¹

Richard Sawyer

A typical and important use of college entrance tests is course placement, i.e., matching students with instruction appropriate to their academic preparation. For example, students whose academic skills are insufficient for them to be successful in a standard first-year English course might, on the basis of their test scores and other characteristics, be advised or required to enroll in a remedial English course. On the other hand, students with an unusually high level of academic preparation might be encouraged to enroll in an accelerated course or in a higher-level course.

Most colleges and universities enroll students who are not academically prepared to do work at a level traditionally expected of their first-year students. The percentage of postsecondary institutions with some form of placement and remedial instruction has steadily increased in the past decade, and is now about 90% (Woods, 1985; Wright and Cahalan, 1985; McNabb, 1990; "Colleges and Universities Offering Remedial Instruction," 1994). A recent survey by the American Council on Education (1996) suggests that about 17 percent of students in community colleges and about 11 percent of students in public four-year institutions take remedial courses. One suggested explanation for the growth of remedial instruction is that American high schools have become less effective in preparing students for college (The National Commission on Excellence in Education, 1983; The Carnegie Foundation for the Advancement of Teaching, 1988; Singal, 1991). Another explanation is that more students from disadvantaged backgrounds are attending college (Munday, 1976; College Entrance Examination Board, 1977; Carriuolo, 1994).

During the past three decades, several authors have proposed using statistical decision theory to validate educational selection systems. Cronbach and Gleser (1965) adapted linear regression methodology to estimate the expected costs and benefits of using a test score or other predictor variable for classifying or selecting personnel. Their technique continues to be widely applied in employment selection. Petersen and Novick (1976) developed a model based on Bayesian decision theory. Ben-Shakhar, Kiderman, and

¹ I am grateful to Dan Anderson and Jerry Dallam for their help in collecting data for this study, and to Mark Houston, Julie Noble, and Jim Scoring for their helpful comments on an earlier draft of this paper.

Beller (1994) compared these two approaches, and illustrated them using data from an admission selection problem.

Sawyer (1996) proposed a statistical decision theory model for validating course placement variables such as test scores. The model can be used to compare the effectiveness of alternative placement variables in identifying underprepared students, and to determine appropriate cutoff scores on these placement variables. Sawyer (1994, 1995, 1996) proposed another, more comprehensive decision theory model for measuring the effectiveness of remedial instruction, and he studied alternative methods for eliciting students' preferences for the possible outcomes in their courses. This study applies the more comprehensive model to first-year mathematics courses at a midwestern university. Elicited preferences for outcomes are combined with actual course outcome data to make inferences about the benefit of prior remedial instruction for subsequent academic achievement.

Background

Remedial Instruction

At many postsecondary institutions, there are two levels of first-year courses: a 'standard' course in which most students enroll, and a 'remedial' course for students who are not academically prepared for the standard course. At some institutions, the lower-level course may be given other names, such as 'college-preparatory', 'compensatory', 'developmental', or 'review'. Carriuolo (1994) articulated differences in the meanings of 'remedial' and 'developmental'. Often, remedial courses do not carry credit toward satisfying degree requirements. At some institutions, there may be courses that require more knowledge and skills than the lowest-level remedial course, but less than the standard course. In the model considered here, only a single lower-level course is considered, and it is designated 'remedial', to be consistent with Willingham's (1974) nomenclature.

Though essential to placement, testing is but one component of an overall system. To be educationally effective, a placement system must satisfy all of the following requirements:

1. Students who have small chance of succeeding in the standard course (underprepared students) are accurately identified.

2. Appropriate remedial instruction is provided to these underprepared students.
3. Both the students who originally enrolled in the standard course, and the students who were provided remedial instruction, eventually do satisfactory work in the standard course.

Note that accurately identifying underprepared students (Requirement 1) is necessary, but not sufficient, for a placement system as a whole to be effective. Accurate identification is not an end, but only a mechanism for effectively allocating remedial instruction (Requirement 2). On the other hand, providing remedial instruction is itself only a means to achieve the larger goal that students succeed in college: Even if underprepared students are accurately identified and are provided remedial instruction, if they eventually drop out or fail in the standard course, then little will have been accomplished by the placement system. On the contrary, both the institution's and the students' resources will have been wasted. Van der Linden (1991) noted that a defining characteristic of course placement systems is that students take different treatments (courses), and the success of each treatment is measured by the *same* criterion variable. Requirement 3 can also be stated in terms of Cronbach's (1971) "aptitude/treatment interaction": The relationship between the placement variables ("aptitudes") and the common criterion variable (success in the standard course) varies according to whether students receive remedial instruction ("treatment").

One might argue that failure in the standard course can lead to positive results for students, such as their selecting and succeeding in another educational program better matched with their talents and interests. While this statement is undoubtedly true for some students, they would have done better to select their preferred educational programs in the first place, through appropriate counseling. This scenario illustrates that effective counseling is important for effective placement. This paper does not, however, attempt to model the effect of counseling on the outcomes of placement.

The need for an institution to serve students who by traditional standards are academically unprepared for college imposes a fourth requirement on placement systems. Even if a large proportion of the underprepared students are accurately identified, are provided remedial instruction, and ultimately succeed in the standard course, the overall result still might not be satisfactory. This would occur if an

institution diverted resources to instruction in the remedial course to such an extent that the performance of students in the standard course was adversely affected. In other words, institutions should consider the tradeoffs they must make in allocating their finite resources when they provide remedial placement systems; such considerations may relate to institutional mission and policy, as much as to costs and to grades. A related concern is that publicly-supported educational institutions must compete for money with agencies that serve other pressing social needs. There is controversy about the proper role of remedial placement in the missions of postsecondary institutions, particularly in publicly-supported institutions. Mac Donald (1994) argued that by overexpanding its remedial programs, the CUNY system seriously degraded the quality of its standard-level undergraduate programs. Lively (1993) reported on efforts in different states to eliminate remedial instruction from four-year public institutions by designating that role to two-year colleges. The American Council on Education (1996) reported on similar efforts in Congress.

Non-cognitive variables in course placement systems

Academic achievement, as measured by persistence and grades in standard courses, is certainly an important outcome of course placement, but it is not the only important outcome. For example, most colleges want their students to achieve other, non-cognitive goals (e.g., working well in groups, understanding and respecting different points of view, developing career and intellectual goals). Furthermore, certain non-cognitive factors (e.g., prior work experience; hours worked on a job while in school; ethnic background) are well-known to be important factors in academic success. For both these reasons, effective course placement requires effective counseling (ideally, one-on-one), and *vice versa*.

Non-cognitive variables are also important in evaluating the effectiveness of a course placement system. For example, do students believe the advice they have been given was appropriate? Do students think that they have been treated well by the system? Do the faculty and staff who run the system believe that their skills are effectively used, and that *their* needs are considered? While issues such as these are beyond the scope of this paper, they need to be addressed in evaluating the overall effectiveness of course placement systems (Frisbie, 1982).

A Decision Theory Model for Course Placement

The decision problem can be formally defined as follows: One must select a particular decision d from a set D of possible decisions. A particular outcome θ , from among a set of possible outcomes Θ , will occur in the future. A utility function $u(d, \theta)$ assigns a numerical value to the desirability of decision d when the outcome is θ . The exact outcome θ that occurs is unknown to the decision maker, but there is some probabilistic information available about the likely values of θ . In a Bayesian decision model, this information is described by a subjective probability distribution on Θ ; the subjective probability distribution quantifies the decision maker's personal beliefs about the likely values of θ , given both prior beliefs and any relevant data collected. The Bayesian optimal strategy is to choose the decision d that maximizes the expected value of $u(d, \theta)$ with respect to the subjective probability distribution on Θ (Lindley, 1972). (See Appendix A for a discussion of expected utility functions.)

To apply this tool to course placement, suppose there is a cutoff score K on a placement test, and that:

- * test scores are obtained for all students for whom a placement decision must be made (i.e., the *placement group*).
- * students whose test scores are less than K are provided remedial instruction before they enroll in the standard course, and students whose test scores are greater than or equal to K enroll directly in the standard course; and
- * the actual final performance in the standard course is eventually known for all students (i.e., for students who are provided remedial instruction, as well as for those who are not).

The final performance in the standard course of students who first enroll in the remedial course will, of course, become known later than the performance of students who enroll directly in the standard course. For each student, twelve possible events could occur, as shown in Table 1 on the following page. Each student is classified according to initial placement and final performance in the standard course. The initial placement is either in the standard course (if the test score equals or exceeds the cutoff score K),

or in the remedial course (if the score is less than K). The final outcome in the standard course is defined by whether the student completed the course, and if so, the final grade.

Table 1
Events Associated with Identifying and Providing Remedial Instruction
to Underprepared Students

Test score	First course into which student is placed	Eventual performance in standard course					
		Does not complete course	Completes course with grade . . .				
			F	D	C	B	A
< K	Remedial	(1)	(2)	(3)	(4)	(5)	(6)
≥ K	Standard	(7)	(8)	(9)	(10)	(11)	(12)

In a more complex (and realistic) model, placement decisions would be based on more than one variable. For example, students whose test scores lie in a decision zone about the cutoff score K might use other information (such as the number of hours they can spend studying, or their other responsibilities) to decide which course to take. The simplified model described by Table 1 illustrates the essential features of the decision theory model.

One could also expand the model by considering "Incompletes" in the standard course. Many postsecondary institutions assign the mark *I* to students who do not complete a course during the academic term in which it is offered, but who believe that they can complete the work during the next academic term. When a student does complete her or his work, the *I* is replaced by the appropriate course grade *A* - *F*. If the student does not complete her or his work in a prescribed time, then the *I* is replaced by an *F*. An "Incomplete" *I* is distinguished from a "Withdrawal" (*W*), in that a *W* does not require a student ever to finish the course. A *W* carries no academic credit, but it is not used to calculate

grade point averages, either. Analyzing "Incompletes" and "Withdrawals" separately in the model would require collecting additional follow-up data. For convenience, I have interpreted "Incompletes" and "Withdrawals" the same (Events (1) and (7) in Table 1).

One could also simplify the model by combining some of the grade categories. For example, the outcomes [Complete course with A], [Complete course with B], and [Complete course with C] could be combined into a single outcome [Successful], and all other outcomes would be considered [Unsuccessful]. I have chosen to use the more complex set of outcomes shown in Table 1, because both students and instructors appear to make meaningful distinctions among the outcomes in terms of their elicited preferences (see *Eliciting Value Functions* below).

Because the events in Table 1 depend on K , the set of decisions (D) in this case is the set of possible values of K . The set Θ consists of the placement test scores and grades in the standard course for all the students in the placement group. The goal is to find the best value of K , and to quantify the effectiveness of the associated instruction. At an institution without a placement system, the events in Table 1 could be observed as follows:

- * Randomly assign students, regardless of their test scores, either to enroll directly in the standard course or to enroll first in the remedial course.
- * Observe the students' eventual performance in the standard course; in either group, some of them will succeed and some will not succeed.

For each value of K , there would be a set of proportions associated with the events (1) - (12). Let us suppose, temporarily, that data are collected this way; the modifications required when there is prior selection resulting from an existing placement system are described in Appendix A.

Let $p_1(K), \dots, p_{12}(K)$, denote observed proportions of students corresponding to the events shown in Table 1 when the cutoff score is K . The overall usefulness of the predictions and the remedial instruction can be evaluated in terms of the proportions and the benefits associated with Events (1) - (12). A function that assigns a value to outcomes such as these is called a utility function.

One class of utility functions is a weighted sum of the proportions $p_1, \dots, p_{12}$:

$$u(K; \theta) = \sum_{i=1}^{12} w_i p_i(K) \quad (1)$$

where $0 \leq w_1, \dots, w_{12} \leq 1$. Such a function would quantify the different costs and benefits of each outcome. Consider, for example, the trade-offs a student must make in his or her utility. Although students pay tuition to take remedial courses (just as they do to take other courses), remedial courses often do not carry college credit. From a student's perspective, the weights $w_1, \dots, w_{12}$ must balance the likely improvement in her or his eventual performance in the standard course against the extra time and money spent on taking the remedial course.

In principle, utility functions are person-specific, and hence need to be elicited separately for each student, counselor, teacher, or administrator. In practice, this is not feasible, and we must look for utility functions that reasonably approximate the preferences of different groups of people.

Expected Utility Functions

A utility function cannot be evaluated for the future group of students for whom placement decisions are to be made, because the outcomes (their test scores and eventual performance in the standard course) are not yet known. In Equation (1), for example, the proportions $p_1(K)$, $p_2(K)$, etc., are not known for a particular group of students before they are tested and complete the standard course. These proportions must instead be estimated in some way from data on past students, under the assumption that future students will be similar to past students.

The *expected utility function* is a formal mechanism for dealing with the uncertainty of outcomes in a decision theory model. It is from the expected utility function that decisions on the effectiveness of a placement system can be made. In Bayesian models, an expected utility for a decision d is the average (expected) value of a utility function $u(d, \theta)$ with respect to the decision maker's subjective probability distribution for the outcomes Θ . In the example previously given,

$$E_\theta[u(K, \theta)] = \sum_{i=1}^{12} w_i E_\theta[p_i(K)] \quad (2)$$

where $E_\theta[p_i(K)]$, $(i=1, \dots, 12)$, are estimated from a past group of students. In the Bayesian model, the expected values of the observed proportions pertain to the decision maker's subjective probability

distribution for students' test scores and course grades. Appendix A contains a technical discussion of how the expected utilities associated with the model in Table 1 can be approximated by applying classical statistical methods to data collected from a past group of students.

Optimal Cutoff Scores

If the expected utility $u'(K) = E_\theta[u(K;\theta)]$ attains a maximum value at some cutoff score K_0 , then using K_0 as a cutoff score will result in a greater expected utility for the group than using any other cutoff score. Furthermore, if K_0 is between the minimum and maximum possible scores on the test or other placement variable, then the effectiveness of the placement system as a whole is supported. On the other hand, if u' is a constant or monotone function, then the appropriateness of the placement variable, the remedial instruction, the standard course, or all three, is called into question. If u' is a constant, then the placement variable may be unable to distinguish between students who would benefit from remedial instruction and those who would not. If u' is increasing, then every student would do better to take the remedial course before taking the standard course. If u' is decreasing, then no student benefits from taking the remedial course. Of course, all of these inferences depend on the validity of the success criterion and of the utility function u .

Let $x_{\min}$ be the minimum possible test score. The quantity $E_\theta[u(x_{\min};\theta)]$ is the *baseline utility* associated with no placement (i.e., when all students enroll in the standard course). The difference $E_\theta[u(K;\theta)] - E_\theta[u(x_{\min};\theta)]$ is an indicator of the potential maximum benefit of the placement system, as compared to no placement system.

Multiple placement variables

The discussion so far has involved using a single cutoff score on a test to make placement decisions. Basing placement decisions on multiple predictors of students' performance in the standard course is more desirable than using only a single placement test:

- * It potentially increases placement accuracy, because it brings more information to bear on placement decisions.
- * It decreases the anxiety some students might have about taking tests.

- * It could allow an institution to exempt from testing students whose characteristics as measured by other placement variables, suggest that they have a high probability of success in the standard course.

I believe that basing placement decisions on multiple predictors is also more typical than using a single test score.

A straightforward way to extend the notion of a cutoff score on a single placement test is to base decisions on a minimum required probability of success in the standard course (i.e., on a *cutoff probability*). The statements made about expected utilities for different cutoff scores also apply to cutoff probabilities.

Summary Utility

Whether they rely on single or multiple predictors of students' performance, many postsecondary institutions give students and their counselors final authority in making placement decisions. Therefore, data from operational course placement systems do not typically show a sharp truncation at a cutoff score or cutoff probability, but a trailing off. Moreover, it would be useful to obtain an estimate of the expected utility of the system not only for alternative cutoff scores or cutoff probabilities, but for the system as it is currently functioning. The summary utility SU provides a means to do this. Essentially, the summary utility is computed by averaging the observed utility values from a past group of students: The observed utility value for each student is computed according to the student's final outcome in the standard course and according to whether the student had previously enrolled in a remedial course. Appendix A contains a technical discussion of how the summary utility can be calculated.

As before, let x_{min} be the minimum observed test score. The difference $SU - E_{\theta}[u(x_{min};\theta)]$ is an indicator of whether the placement system as currently operating is of any benefit as a whole, or whether the institution would be better off without the current placement system.

Informal Indicators of Remedial Effectiveness

The expected utility function $E_{\theta}[u(K;\theta)]$ and the summary utility SU are formal indicators of the effectiveness of the placement system as a whole. An institution may also be interested in answering the simpler question, "Do our underprepared students benefit from taking the remedial course, in the sense

that they are more likely to succeed in the standard course?" This question ignores the costs associated with providing remedial instruction and focuses only on the benefits. If the benefits are non-existent or minimal, however, then any consideration of costs will likely return a negative answer with regard to expected utility.

One simple way to indicate the effectiveness of remedial instruction is to compare the conditional probability of success function for students who have previously received remedial instruction with that of students who have not previously received remedial instruction. Let $\hat{P}_R[Y \geq g | X=x]$ be the conditional probability that a student with prior remedial instruction (R) earns a grade Y of g or higher in the standard course, given the student's placement test score x. For example, the value $g=3$ would be associated with the conditional probability of completing the standard course with a B or higher grade. Let $\hat{P}_S[Y \geq g | X=x]$ be the corresponding conditional probability for a student who directly enters the standard course (S) without prior remedial instruction. By overlaying the graphs of $\hat{P}_R[Y \geq c | X=x]$ and $\hat{P}_S[Y \geq c | X=x]$ for the values of x associated with the underprepared students, one can get a visual impression of the benefits of remedial instruction. Clearly, we would want $\hat{P}_R[Y \geq c | X=x] \geq \hat{P}_S[Y \geq c | X=x]$.

One can also compare the estimated marginal distributions of Y according to whether students receive prior remedial instruction:

$$\begin{aligned}\hat{h}_S(y) &= \sum_x \hat{P}_S(y | x) f(x), \\ \hat{h}_R(y) &= \sum_x \hat{P}_R(y | x) f(x)\end{aligned}\tag{2}$$

where $f(x)$ is the marginal probability of a test score x, and the summations range over the test scores x of the students who received remedial instruction. Overlaying graphs of the distributions $\hat{h}_S$ and $\hat{h}_R$ would suggest the extent to which taking the remedial course benefits underprepared students. One could also compute summary statistics (e.g., the mean) for these two distributions.

Retention Indicators

An institution can also consider longer-term indicators of the effectiveness of a remedial course placement system, namely, retention. Institutions have a direct financial incentive, as well as an educational motive, to increase their retention rates: Students who continue to enroll in college continue to pay tuition. From this perspective, accounting approaches could be used to evaluate a course placement system.

Let $E_s(t)$ be the proportion of students who enroll only in standard courses during their first year at the institution, and who remain enrolled at the institution through Year t . Let $E_R(t)$ be the proportion of students who take one or more remedial courses during their first year, and who remained enrolled through Term t . Then,

$$\begin{aligned} P[E_s(t) \leq g] &= \sum_x P[E_s(t) \leq g | x] f(x) \\ P[E_R(t) \leq g] &= \sum_x P[E_R(t) \leq g | x] f(x). \end{aligned} \quad (3)$$

Once appropriate retention data have been collected, the retention probabilities $P[E_s(t) \leq g | x]$ and $P[E_R(t) \leq g | x]$ can be estimated with logistic regression models like that in Equation (A-4) in Appendix A. In principle, one could also model the retention rates of students who take *particular* remedial courses $C_1, \dots, C_k$, or combinations of remedial courses (e.g., C_2, C_4 , and C_6), and compare these rates to the retention rate of students who take no remedial courses.

Furthermore, one could define criterion variables based on both retention and cumulative GPA, in a manner analogous to the variable Y previously discussed. One could also define a criterion variable specifying the number of years students require to graduate (e.g., 4, 5, 6, 7 or more, never); or, one could define a criterion variables based on the number of years required to graduate and on cumulative GPA. Going further, one could measure students' satisfaction with the quality of their college education, or their salaries after they graduate. In principle, one could also elicit utilities for different combinations of values of these criterion variables.

One should expect that any of these alternative criterion variables will be less strongly related to placement test scores and to remedial instruction than criterion variables defined by performance in

particular first-year standard courses. The reason is that over time other variables completely unrelated to students academic skills in particular subject areas (e.g., emotional stability, family finances, physical health) could strongly affect students' overall performance in college.

Murtuza & Ketkar (1995) studied a course placement and advising program at an urban university for its effect on retention and its cost-effectiveness. Cost-effectiveness was determined by a *break-even analysis*: Does the program increase retention enough so that the resulting extra tuition income offsets the program's cost? When they compared recent retention rates to those observed before the program began, Murtuza and Ketkar found that the program was cost-effective; but their analysis of data from recent years produced the opposite result. Murtuza and Ketkar also found that a centralized program (in which staff were hired and assigned to work specifically on course placement and advising) was more cost-effective than a decentralized program (in which these functions were assigned to faculty members).

Analyses such as these can bring important information to decision makers about the effectiveness of course placement systems. I have not yet collected the retention data needed to estimate such models, but plan to do so in the future.

Eliciting Utility Functions

If the decision model is to be useful in real applications, its utility function must accurately describe the preferences of the decision makers. In the model described by Table 1, for example, we need some way to quantify students' and instructors' preferences for success in the standard course, as balanced against the extra time and cost associated with taking the remedial course.

There is a vast literature on eliciting (i.e., assessing) utility functions. One important characteristic distinguishing various utility theories is whether they are deterministic or stochastic:

- * A *value function* measures the satisfaction of any sort of 'want' without regard to uncertainty. For example, some economists model the satisfaction that an individual receives from consuming commodities. The key characteristic of a value function is that it assigns numerical values to the subjective worth of outcomes without regard to uncertainty (Yates, 1990).

A simple example of eliciting a value function would be to ask an individual to rank each possible outcome on the following Likert scale:

1='dislike very much', 2='dislike somewhat', 3='like somewhat', 4='like very much'

Note that in this example, the assignment of values to outcomes is done outside any context of uncertainty or risk.

- * A *von Neumann-Morgenstern* utility, in contrast, is explicitly defined in terms of uncertainty. The standard assumption in von Neumann-Morgenstern (abbreviated hereafter as vN-M) theories is that the decision maker has a preference relation $\prec$ over the set Π of *probability distributions* on the outcome space Θ (rather than on Θ itself), and that $\prec$ satisfies an appropriate set of axioms (e.g., transitivity). Then it can be shown that there exists a real function u on Θ , such that for distributions $p, q \in \Pi$, $p \prec q$ if, and only if, $E_p[u] < E_q[u]$. The function u is unique up to positive, linear transformations; therefore, one can without loss of generality assign the value 0 to the least favorable outcome and the value 1 to the most favorable outcome. Note that vN-M utility functions are defined in terms of probability; therefore, their elicitation is naturally done in reference to hypothesized probability distributions (typically, hypothetical lotteries). See Farquhar (1984) for a comprehensive review of different strategies for eliciting vN-M utility functions.

The principal advantage of value functions is that they are easy to elicit, because they do not require any reference to uncertainty or risk. The principal advantage claimed for vN-M utility functions is that they are more realistic, because they reflect the decision maker's feelings about both the inherent worth of the outcomes, and the risk involved in making choices. On the other hand, this realism is elicited in the context of hypothetical situations. Although both value functions and vN-M utility functions can formally be used in expected utility models (Yates, 1990), they are not the same, and can lead to different decisions. I shall follow Yates' convention in reserving the term *utility* to refer specifically to a vN-M utility function, and the term *value function* for a function that does not consider risk.

Sawyer (1995) elicited vN-M utility functions and value functions from students enrolled in either of two particular postsecondary institutions in the midwest, using a paper-and-pencil questionnaire. He found that about half of the university students and about two-thirds of the community college students surveyed completed the questionnaire. Of students and instructors who completed the questionnaire, about 2/3 provided enough information to develop an internally consistent grade value function. For the typical respondent, only 35% of the possible vN-M utility functions could be calculated, however. These results suggest that institutions can not realistically expect to elicit utilities by administering paper-and-pencil questionnaires to their instructors and students. Institutions would instead need to provide special instruction and motivation to elicit valid utilities for most students and instructors. A computerized interactive system for eliciting utilities (Issacs & Novick, 1978) would also increase validity by giving respondents an opportunity to correct inconsistencies. The potential difficulty and expense of developing and operating such a system could make it unattractive to institutions, however.

Sawyer (1995) found that utility functions elicited from both students and instructors were substantially larger than their elicited value functions. Median elicited utility functions for the grades of B, C, and D, for example, were .90, .75, and .48, respectively. The corresponding median elicited value functions were .80, .50, and .10. Sawyer also found a small but consistent difference between instructors and students in the values of their elicited value functions: Instructors tended to place somewhat greater value on the grades B, C, and D (by about .05 - .10) than did the students. This latter result is consistent with Whitney's (1989) observation that students and teachers have different interests in course placement.

Because value functions are much easier to elicit from a paper-and-pencil questionnaire than are utility functions, I chose to consider value functions exclusively in this study.

Data

Eliciting Value Functions

The value function data for this study came from students enrolled in six elementary mathematics courses at a public postsecondary institution ('Midwestern University') in spring 1997. A professor supervised 12 graduate student instructors of the courses. The professor and the instructors agreed to

administer the questionnaires in class (see Appendix B). Students indicated their relative preferences for the grades A, B, C, D, F, and W (withdrawal) in the standard course by marking these letters on a number scale. The number scale ranged from 0 to 100, in units of 10. Table 2 below summarizes the number of responses by course.

Table 2.
Response Rates to Value Function Elicitation Survey, by Course

Course	Enrollment	Number of completed questionnaires	Response rate
Students			
Course A: Basic Algebra I	25	24	.98
Course B: Basic Algebra II	101	82	.82
Course C: Geometry	7	5	.71
Course E: Trigonometry	74	60	.81
Course I: Elementary Functions	113	66	.58
Course L: Calculus I (1 sec.)	36	31	.86
*** Total ***	356	268	.75

The response rates varied by course, from .58 to .98. The overall response rate (.75) was much higher than the .40 response rate obtained by Sawyer (1996). The reason is that the present questionnaire was administered in-class, rather than taken home.

The questionnaire also directed students to state whether they had previously taken any other mathematics courses at Midwestern University (including the course in which they were currently enrolled). I analyzed separately the responses of students according to their previous course work, to infer the effect of remedial course work on the value functions. I inferred the effect of previous course work, by asking the students who had *not* previously taken mathematics courses to state their preferences

for grades as if they had taken such courses. The questionnaire coached the students that a particular grade under the second scenario would likely not be as satisfying as the same grade obtained without previous remedial course work.

I created a separate version of the questionnaire for each of the six courses listed in Table 2. Appendix B contains the version of the questionnaire for the Calculus I course.

Course outcomes

Figure 1 on the following page shows the mathematics courses typically taken by first-year students at Midwestern University. I have annotated some of the course names with descriptive titles that are commonly used at other institutions. The mathematics faculty advises students to enroll in courses on the basis of their high school course work and their scores on a locally-developed and administered placement test. The mathematics faculty does not, however, enforce the recommendations based on these placement variables.

The flow lines in the diagram, followed from top down, show the sequences in which students typically take different courses. Students who enroll in particular lower-level courses may subsequently enroll in more advanced courses, as indicated by the flow lines. For example, students who enroll in Course I (Elementary Functions) may do so in three different ways:

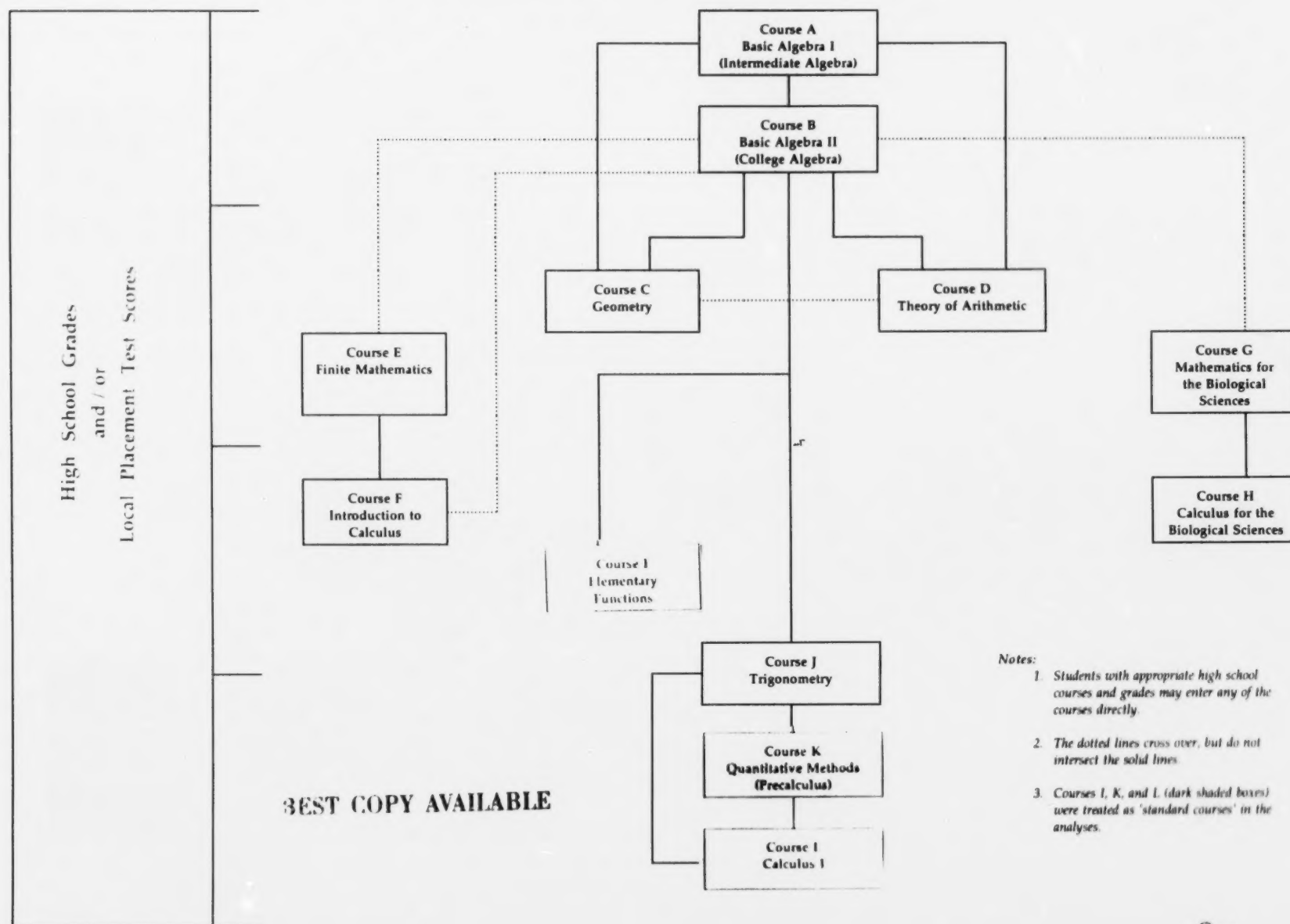
- a. Directly, on the basis of their high school course work and placement test scores.
- b. Initially in Course B, then in Course I.
- c. Initially in Course A, then in Course B, then in Course I.

For other courses there are many more entry paths.

I analyzed grade data for Course I (Elementary Functions), Course K (Quantitative Methods), and Course L (Calculus). Each was treated as a standard course. I allowed Courses I and K, however, to be considered remedial courses in the analyses for which they were not standard courses.

Local placement test score data at Midwestern University are kept in a different office than are course grades. Collecting local test scores, therefore, would have required a separate data collection effort. Because ACT Mathematics scores (ACT, in press) were conveniently available for most students, I elected to treat the ACT Mathematics test as the placement test, even though it is not actually used for this purpose at Midwestern University.

Figure 1.
Structure of Course Placement and Prerequisites
for First-Year Mathematics Courses at Midwestern University



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I obtained from administrators at Midwestern University the ACT Mathematics scores, course enrollment histories, and course grade data for all students who enrolled in any of the courses A - L during the years 1993, 1994, or 1995. From these data, I was able to determine the exact sequence of courses of students who eventually enrolled in Courses I, K, or L. To maximize the amount of data from the lower-level courses, I restricted the analyses to students who enrolled in Courses I, K, or L in 1994 or 1995.

Analyses

Questionnaire data

For each student, I computed a value function for the 12 outcomes described in Table 1. The value function $v(r,g)$ measures the student's preferences for different grades g in the standard course, given the presence ($r=1$) or absence ($r=0$) of prior remedial instruction:

- a. For students who had not previously taken mathematics courses at Midwestern University, let $v(0,g)$ denote the responses marked in Question 1, and let $v(1,g)$ denote the responses marked in Question 3. (Questions 1 and 3 ask students to mark number lines; see Appendix B.)
- b. For students who had previously taken mathematics courses at Midwestern University, let $v(1,g)$ denote the responses marked in Question 1, and let $v(0,g)$ be undefined.

I also determined the *coherence* of the elicited value function v . The value function was deemed to be coherent if:

$$\begin{aligned} v(0,A) &\geq v(0,B) \geq v(0,C) \geq v(0,D) \geq \min\{v(0,W), v(0,F)\}, \\ v(1,A) &\geq v(1,B) \geq v(1,C) \geq v(1,D) \geq \min\{v(1,W), v(1,F)\}, \text{ and} \\ v(0,g) &\geq v(1,g), \text{ for } g = A, B, C, D, F, W. \end{aligned}$$

Let $CHR=1$, if v is coherent, and $CHR=0$, if v is not coherent. I computed a frequency distribution for CHR , and summarized the distribution of v (using cases for which it was coherent), given students' previous instruction in mathematics.

Course grade data

I summarized, by course and by prior remedial instruction, the distribution of ACT Mathematics scores and course grades, and calculated estimates of the conditional probability of completing the course with a B or higher grade, given ACT Mathematics score and prior remedial instruction. I also calculated analogous estimates of the conditional probability of completing the course with a C or higher grade.

From the ACT Mathematics scores and end-of-term course grades, I estimated the conditional probability of success functions $P_S(y|x)$ and $P_R(y|x)$ according to the discussion of the equations in Equation (A-5). I computed estimates separately, using the assumptions of common slopes and separate slopes (see discussion in Appendix A). I then estimated for each course the marginal distribution of course grades, by prior remedial instruction (Equation A-10), and compared the two estimated distributions.

I next estimated expected value functions for each course. For the cell probabilities $p_i(K)$ in Equation (A-2), I used the estimated conditional probabilities P_S and P_R ; for the weights w_i , I used the medians of the value functions of the group of students who had not previously taken mathematics courses. I also calculated for each course the summary utility SU and compared it to $E_\theta[u(x_{min};\theta)]$ (see Equation (A-9)).

Results

Value Functions

Table 3 on the following page summarizes the coherence indicator CHR by students' prior remedial instruction. The coherence results for students who had previously taken remedial instruction in mathematics pertain only to Question 1 of the survey instrument. The coherence results for students who had not received remedial instruction in mathematics are presented both for Question 1 (which pertains to their current actual course) and for Question 3 (which pertains to their current course, under the hypothetical assumption that they had received prior remedial instruction).

Only about 56% of the students who received prior remedial instruction expressed value functions that were coherent for all the grades B, C, D, F, and W. Most of the incoherent value functions resulted

Table 3.
Proportions of Respondents with Coherent Value Functions,
by Student Group

Student group	N	Current actual course		Current course after hypothetical previous course		Current actual course & current course after hypothetical previous course	
		Grades B,C,D,F,W	Grades B,C,D,F	Grades A,B,C,D,F,W	Grades B,C,D,F	Grades A,B,C,D,F,W	Grades B,C,D,F
Students with prior remedial instruction	132	.56	.83	...	...	...	...
Students with no prior remedial instruction	136	.52	.88	.27	.64	.23	.50

from missing values for the grade W: About 83% of the students had coherent value functions, except for missing values of W.

Only 52% of students who had not previously taken remedial instruction expressed value functions that were coherent for all the grades B, C, D, F, and W in Question 1; only 27% expressed value functions that were coherent for all the grades A, B, C, D, F, and W in Question 3; and only 23% expressed value functions that were coherent for both Question 1 and Question 3. Most of the incoherent value functions resulted from missing values for the grade W in Question 1, or for the grades A and W in Question 3. Except for the grades A and W, 88% of students who did not receive prior remedial instruction had coherent value functions for Question 1; 64% had coherent value functions for Question 3; and 50% had coherent value functions for both Questions 1 and 3.

In an earlier effort to construct value functions from students' responses to questionnaires (Sawyer, 1996), I obtained a coherence rate of .77. The earlier results were, however, based on a different questionnaire and methodology.

To increase the size of the samples on which the statistics were computed, I imputed value functions for the grades A and W for students who did not provide value functions for these grades, but whose value functions were otherwise coherent. Among the students with completely coherent value functions, the mean value for W exceeded the mean value for F by about 1/3 of the difference between the mean value for D and the mean value for F. Furthermore, the mean value for A (under the assumption of prior remedial instruction) exceeded the mean value for B by about .98 of the difference between 1 and the mean value for B. Therefore, I imputed a value function v' as follows:

$$\begin{aligned} v'(r,W) &= v(r,W), \text{ when } v \text{ was coherent, and} \\ &= v(r,F) + [v(r,D) - v(r,F)]/3, \text{ when } v(r,W) \text{ was missing, but } v \text{ was otherwise coherent,} \end{aligned}$$

$$\begin{aligned} \text{and } v'(1,A) &= v(1,A), \text{ when } v \text{ was present and coherent, and} \\ &= v(1,B) + 0.98*[1 - v(1,B)], \text{ when } v(1,W) \text{ was missing, but } v \text{ was otherwise coherent.} \end{aligned}$$

Table 4 on the following page shows the medians and interquartile ranges of the imputed value functions. As in Table 3, the results are presented separately according to students' prior remedial

Table 4.
Medians (Inter-Quartile Ranges) of Imputed Value Functions, by Student Group

Student group	Current actual course							Current course after hypothetical previous course						
	N	A	B	C	D	W	F	N	A	B	C	D	W	F
1. Students with prior remedial instruction	110	1.00 (0.00)	0.80 (0.15)	0.50 (0.30)	0.10 (0.25)	0.03 (0.10)	0.00 (0.00)	...	...	...	...	...	...	...
2. Students with no prior remedial instruction	68	1.00 (0.00)	0.70 (0.20)	0.30 (0.30)	0.10 (0.20)	0.03 (0.10)	0.00 (0.00)	68	0.99 (0.01)	0.53 (0.28)	0.20 (0.30)	0.00 (0.10)	0.00 (0.03)	0.00 (0.00)

instruction in mathematics. The students who had previously taken remedial instruction in mathematics (Group 1) appeared to value the grades of B and C more than students who had not received remedial instruction (Group 2): The medians of $v'(B,1)$ and $v'(C,1)$ were 0.80 and 0.50, respectively, for the former group; and the means of $v'(B,1)$ and $v'(C,1)$ were 0.53 and 0.20 respectively, for the latter group. The medians for the grades D, F, and W were nearly the same for the groups. Among both groups the mean for W exceeded by a small amount the mean for F, but the medians were the same.

The students who had not previously received remedial instruction (Group 2) clearly expressed a disutility (loss in value) under the assumption that they had taken a remedial course. For example, the median of $v'(B,0)$ for the current actual course was 0.70, but the median of $v'(1,B)$ was 0.53 under the hypothetical assumption that they had taken a prior remedial course. The students also indicated disutility in the other grades under this assumption.

Here is the ranking of the twelve possible outcomes (r, g) , where $r=0, 1$; and $g=A, B, C, D, W, F$ according to the median value functions in Row 2 of Table 4:

$$(0,A) > (1,A) > (0,B) > (1,B) > (0,C) > (1,C) > (0,D) > (0,W) > (0,F) = (1,D) = (1,W) = (1,F),$$

where the last three outcomes have zero value. Sawyer (1996), using a different questionnaire, obtained similar rankings, except for the outcomes $(1,A)$ and $(0,B)$. In the earlier study, Sawyer found that $(0,B) > (1,A)$.

Course Placement Summary Statistics

Table 5 on the following page shows for each course the sample size, mean and standard deviation of ACT Mathematics score, and the mean and standard deviation of the end-of-term grade for Courses I, K, and L. The mean and standard deviation for the course grade were based on data from students who completed the course. The last column of Table 5 shows the percentage of students who completed the course.

Table 5.
Summary Statistics for ACT Mathematics Score and End-of-Term Grade,
by Course

Course	Group	N	ACT Mathematics score		End-of-term grade		
			Mean	SD	Mean (A-F)	SD (A-F)	Pct. completed course
Course I: Elementary Functions	With prior rem. instr.	17	18.1	4.6	1.75	1.20	77
	No prior rem. instr.	562	23.1	3.7	2.30	1.23	86
Course K: Quantitative Methods	With prior rem. instr.	343	20.5	3.3	2.22	1.05	88
	No prior rem. instr.	1943	23.6	3.4	2.51	1.11	91
Course L: Calculus I	With prior rem. instr.	126	23.1	3.9	2.24	1.12	84
	No prior rem. instr.	1082	25.9	3.7	2.21	1.07	87

Course I (Elementary Functions)

Table C-1 in Appendix C summarizes the logistic regression of success on ACT Mathematics score, by course and prior remedial instruction. Results are given for two definitions of 'success': completing the course with a B or higher grade, and completing the course with a C or higher grade. Figure 2 (on a separate page following this discussion) shows the relationship between probability of success (B or higher) in Course I (Elementary Functions) and ACT Mathematics score. The curve and bars plotted in black pertain to students who received prior remedial instruction, and the curve and bars plotted in white pertain to students who did not receive prior remedial instruction. Figure 2 shows that among students with prior remedial instruction, ACT Mathematics score was essentially unrelated to the probability of completing the course with a B or higher grade. For students who entered Course I directly, without prior remedial instruction, ACT Mathematics score was strongly related to the end-of-term grade in Course I. A similar result occurred for the C-or-higher success criterion (not shown in Figure 2).

Tables C-2 and C-3 in Appendix C contain results of the logistic regression analyses of the dichotomous variables $T_1, T_0, \dots, T_3$ defined on p. A5 in Appendix A. Table C-2 shows the results under the assumption of equal slopes for the ACT Mathematics score in the five models. Table C-3 shows the results under the assumption of separate slopes for the ACT Mathematics score in the five models. Only 17 students in Course I had prior remedial instruction; this sample size was obviously inadequate to estimate the logistic regression coefficients. In the equal slopes model, the estimated standard errors were all larger than the estimated coefficients, and the situation was even worse for the separate slopes model. Therefore, the following discussion of the marginal distribution of grades, expected value function, and summary utility for Course I should be considered only illustrative, rather than indicative of the true state of affairs:

- Figure 3 shows the estimated distribution of grades in Course I received by students with prior remedial instruction (black columns). This distribution is actually slightly worse than the distribution of grades that would be projected if the students had not taken remedial courses.

- The summary utility for students in the current placement system was .41, as compared to a baseline utility of .42. This result suggests that remedial instruction was ineffective in increasing the expected value function.
- Finally, the expected value function in Figure 4 is flat over ACT Mathematics scores of 17 or less, and then declines monotonically. This result also suggests that prior remedial instruction was ineffective in increasing the expected value function.

If these statistics had been based on adequate samples, they would have all pointed to the conclusion that prior remedial instruction was not effective in helping students succeed in Course I.

Conditional Probability of B or Higher Grade in Course I,
and Distribution of ACT Mathematics Score,
by Prior Remedial Instruction

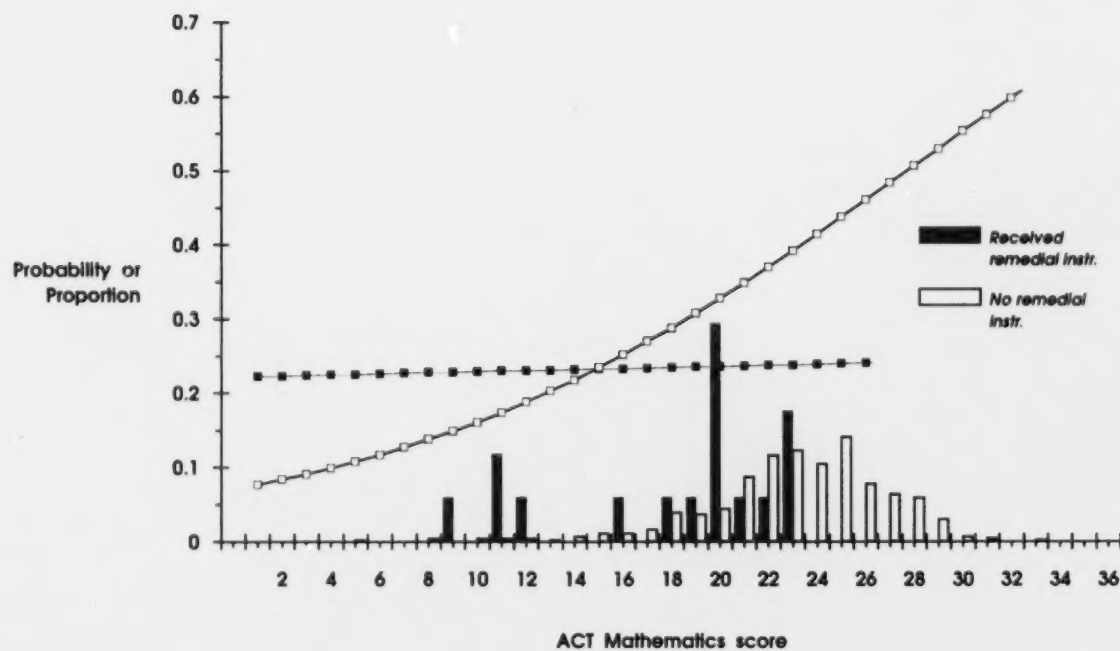


Figure 2.

Estimated Distribution of End-of-Term Grades in Course I,
by Prior Remedial Instruction

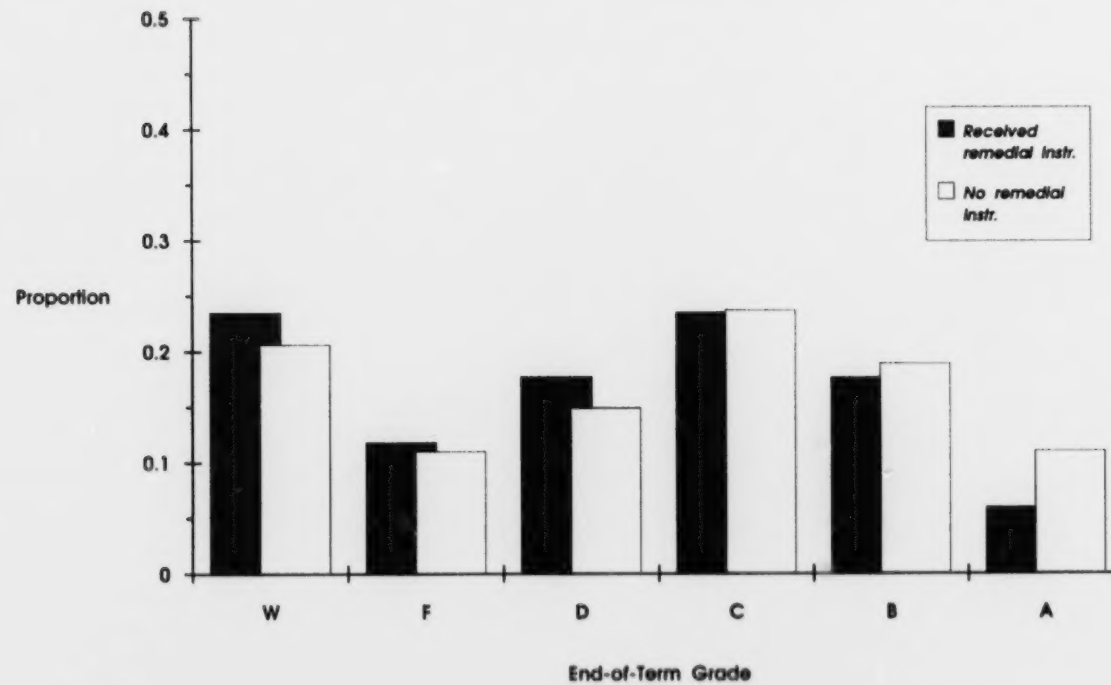


Figure 3

Expected Value Function for Course I,
by ACT Mathematics Score

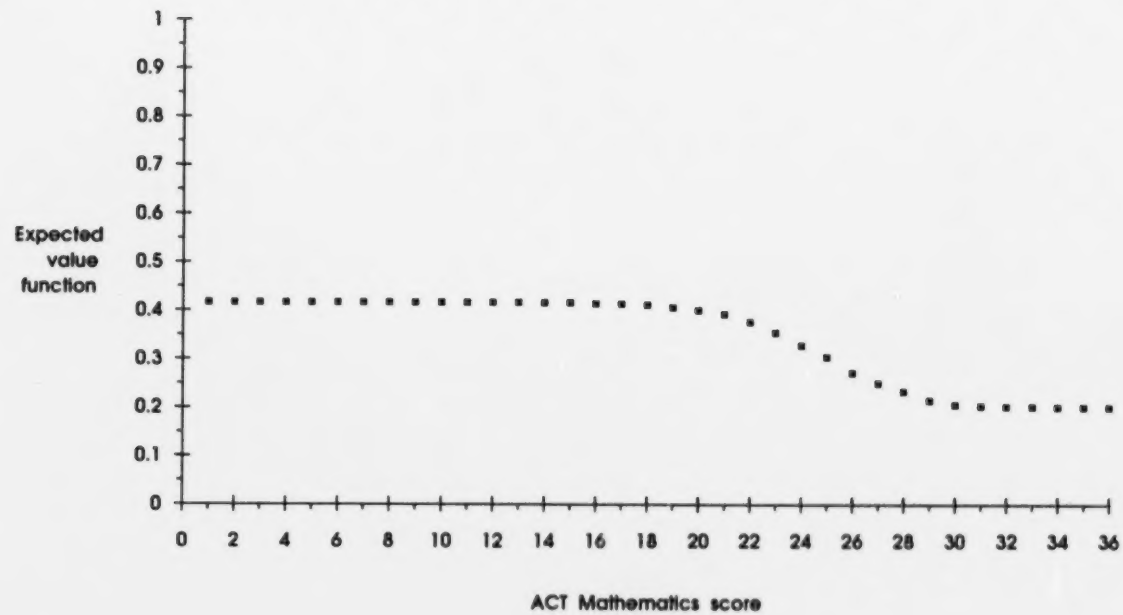


Figure 4.

Course K (Quantitative Methods)

Table C-1 contains statistics pertaining to the estimated logistic regression of success on ACT Mathematics score and prior remedial instruction. Figure 5 shows the results for the B-or-higher success criterion. ACT Mathematics score was positively related to success in Course K among both the students who had previously received remedial instruction and among those who enrolled directly in the standard course. For ACT Mathematics scores less than 23, taking remedial course work increased the probability of success. Although the fitted curves suggest that for ACT Mathematics scores greater than 23, taking the remedial course work was associated with a lower probability of success, the histograms show that there were few such students.

The benefit of remedial course work to underprepared students is also apparent in Figure 6. Note that students who previously took remedial instruction were less likely to receive grades of W and F, and more likely to receive grades of D, C, or B. The difference in proportions for individual grades was less than .05.

Unfortunately, the benefit of taking remedial instruction was apparently not great enough to compensate for the loss in utility in doing so. Figure 7 shows that the expected value function curve is flat over ACT Mathematics scores of 16 or less, and then declines monotonically. The sample sizes associated with the logistic regressions of the dichotomous variables $T_1, \dots, T_3$ are large enough so that all the estimated coefficients have small estimated standard errors. Therefore the result shown in Figure 7 is not likely due to chance.

There was a similar result for the summary utility. The summary utility for students in the current system was .46, as compared to a baseline utility of .47.

The regression weights for the dichotomous variables $T_1, \dots, T_3$ in the separate-slopes model (see p. A3) were all statistically significant. The differences in the regression weights for ACT Mathematics score were also statistically significant. The differences were small enough, however, for the equal-slopes model to be a reasonable approximation; the estimated weights ranged from -.17 to -.26.

Conditional Probability of B or Higher Grade in Course K,
and Distribution of ACT Mathematics Score,
by Prior Remedial Instruction

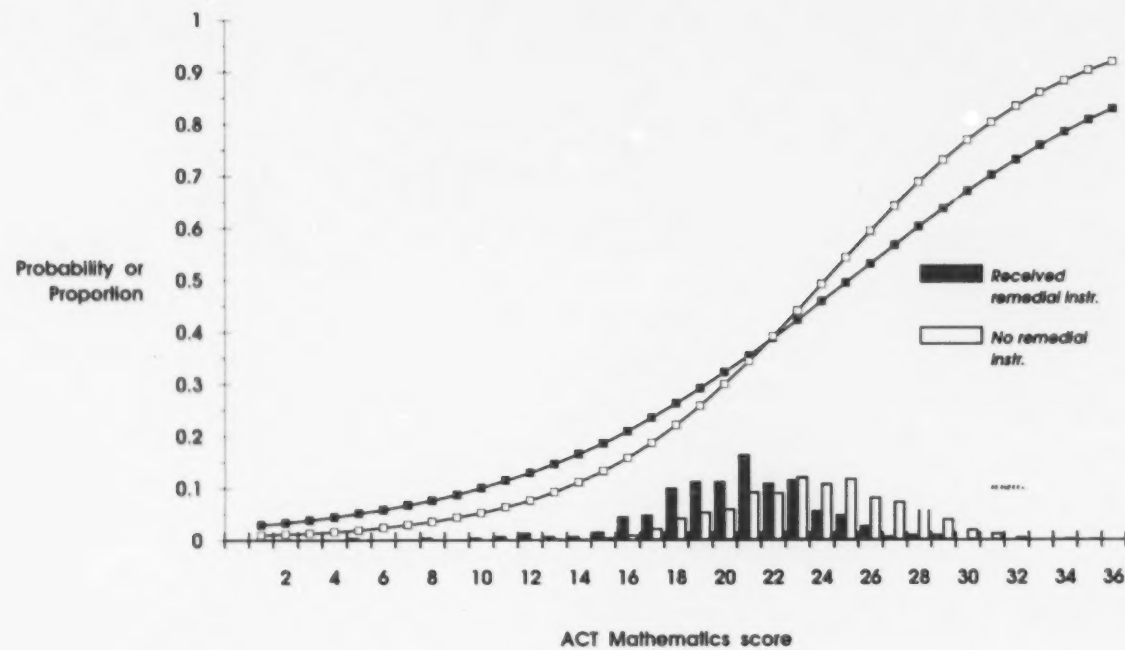
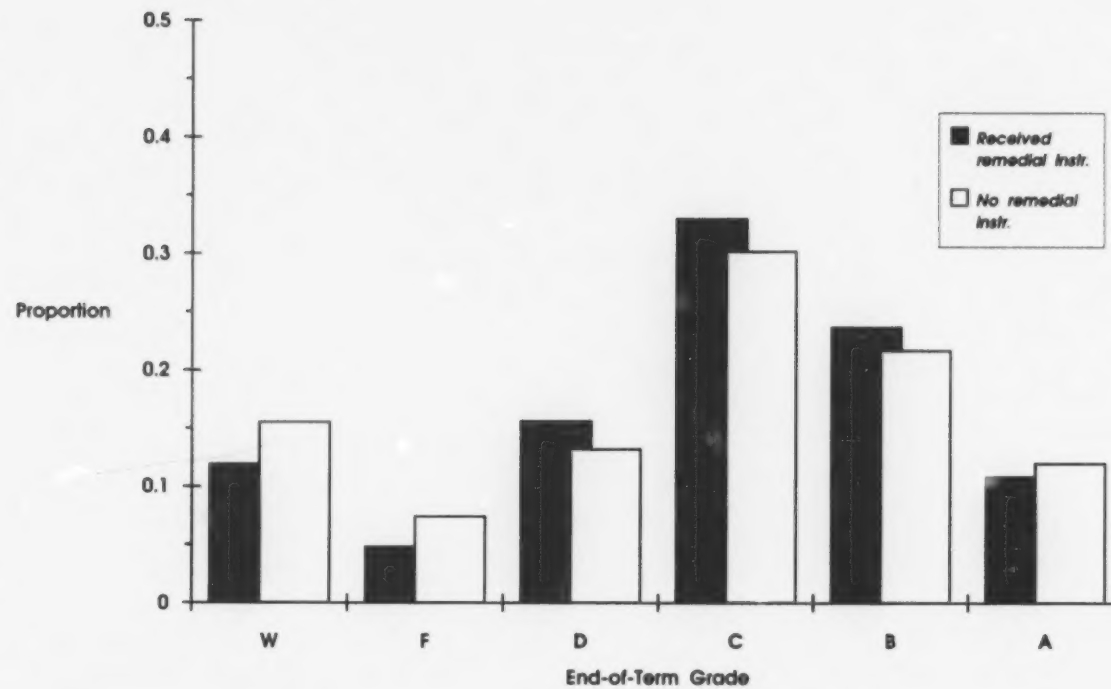


Figure 5.

Estimated Distribution of End-of-Term Grades in Course K,
by Prior Remedial Instruction



Expected Value Function for Course K,
by ACT Mathematics Score

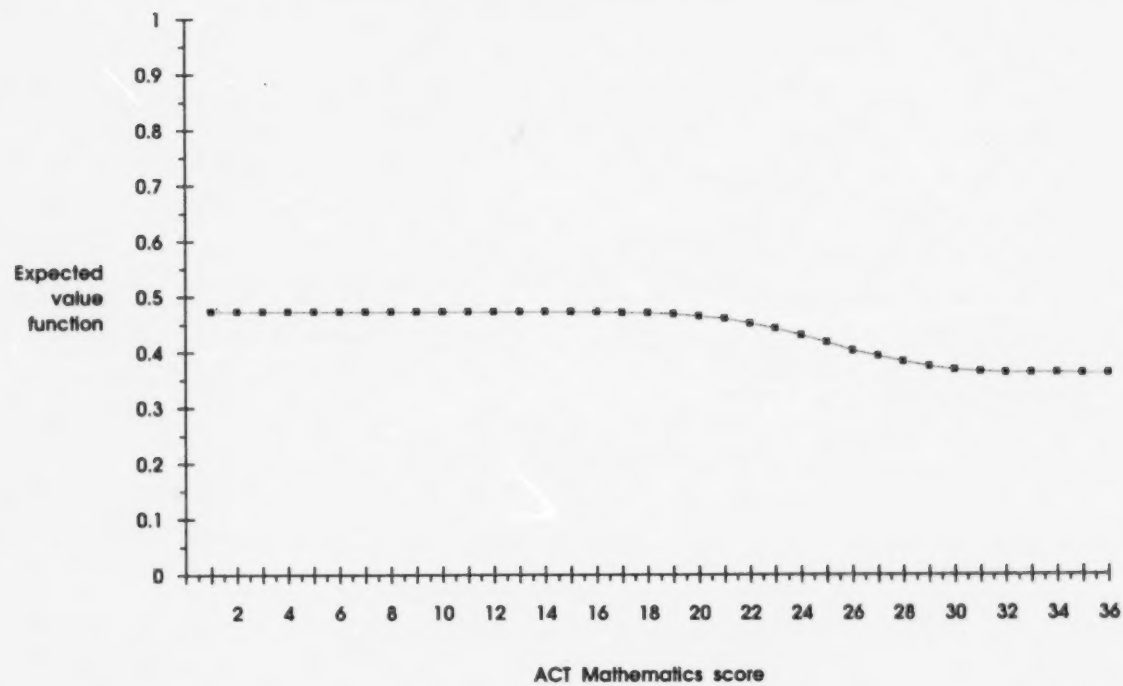


Figure 7.

Course L (Calculus)

Figure 8 shows that ACT Mathematics score was positively related to success (B or higher) in Course L for both the students who received remedial instruction and those who enrolled directly in the standard course. (The ACT Mathematics score regression weight for students who received remedial instruction was not statistically significant, however; see Table C-1.) For all ACT Mathematics scores less than 25, taking remedial course work increased the probability of success. The fitted curves suggest that for ACT scores greater than 25, taking remedial course work was associated with a lower probability of success; but, the histograms show that there were few such students.

The benefit of remedial course work to underprepared students is also apparent in Figure 9. Note that students who had previously taken remedial instruction were less likely to receive grades of W and F, and more likely to receive grades of D, C, or B.

Unfortunately, as in Course K, the benefit of taking remedial instruction was not great enough to compensate for the disutility in doing so. Figure 7 shows that the expected value function curve is flat over ACT Mathematics scores of 23 or less, and then declines monotonically. The sample sizes associated with the logistic regressions of the dichotomous variables $T_1, \dots, T_3$ for students with prior remedial instruction were not large enough for the estimated intercepts to be statistically significant.

There was a similar result for the summary utility. The summary utility for students in the current system was .48, as compared to a baseline utility of .49.

The regression weights for the dichotomous variables $T_1, \dots, T_3$ in the separate-slopes model were all statistically significant. The differences in the regression weights for ACT Mathematics score were also statistically significant. The differences were small enough, however, for the equal-slopes model to be a reasonable approximation; the estimated weights ranged from -.10 to -.27.

**Conditional Probability of B or Higher Grade in Course L,
and Distribution of ACT Mathematics Score,
by Prior Remedial Instruction**

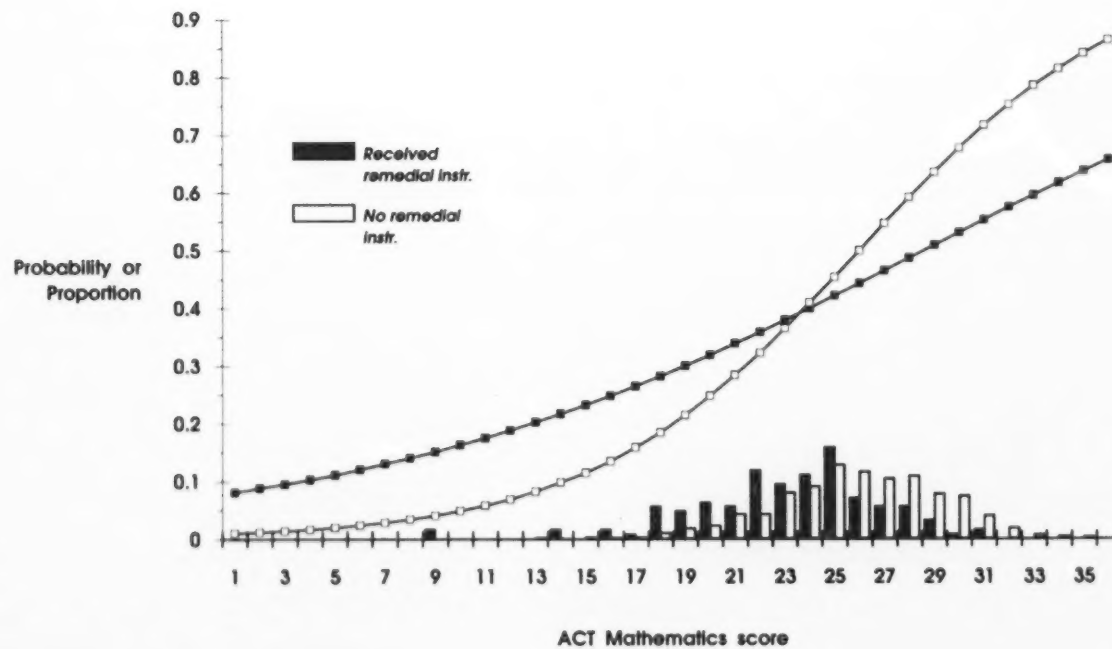
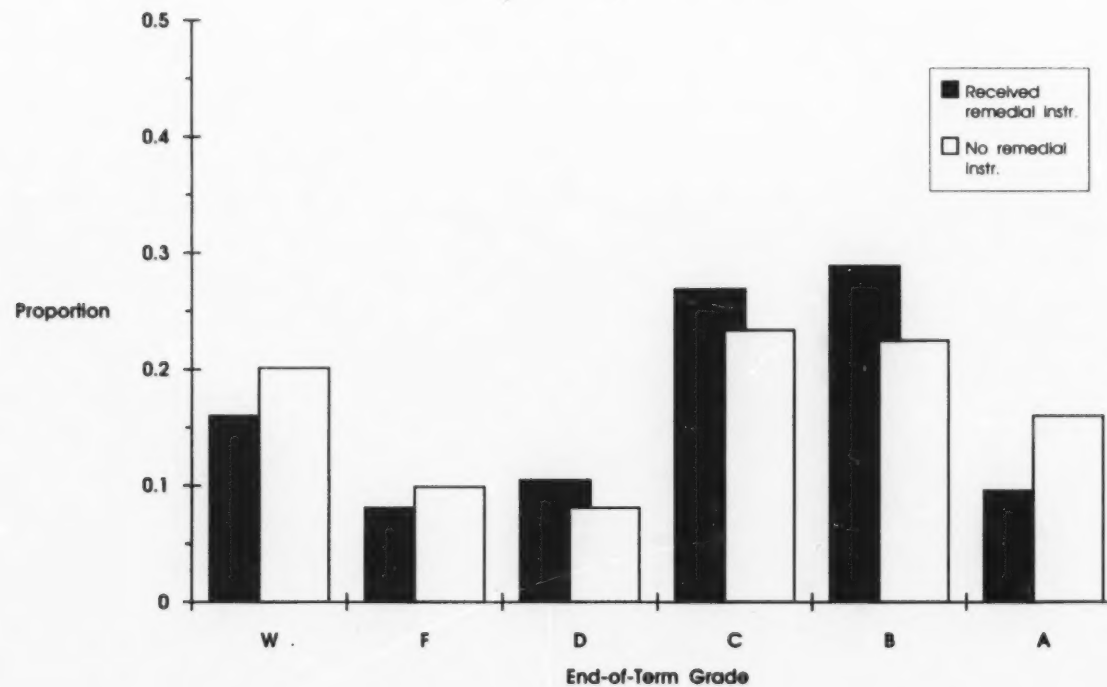


Figure 8.

Estimated Distribution of End-of-Term Grades in Course L,
by Prior Remedial Instruction



Expected Value Function for Course L,
by ACT Mathematics Score

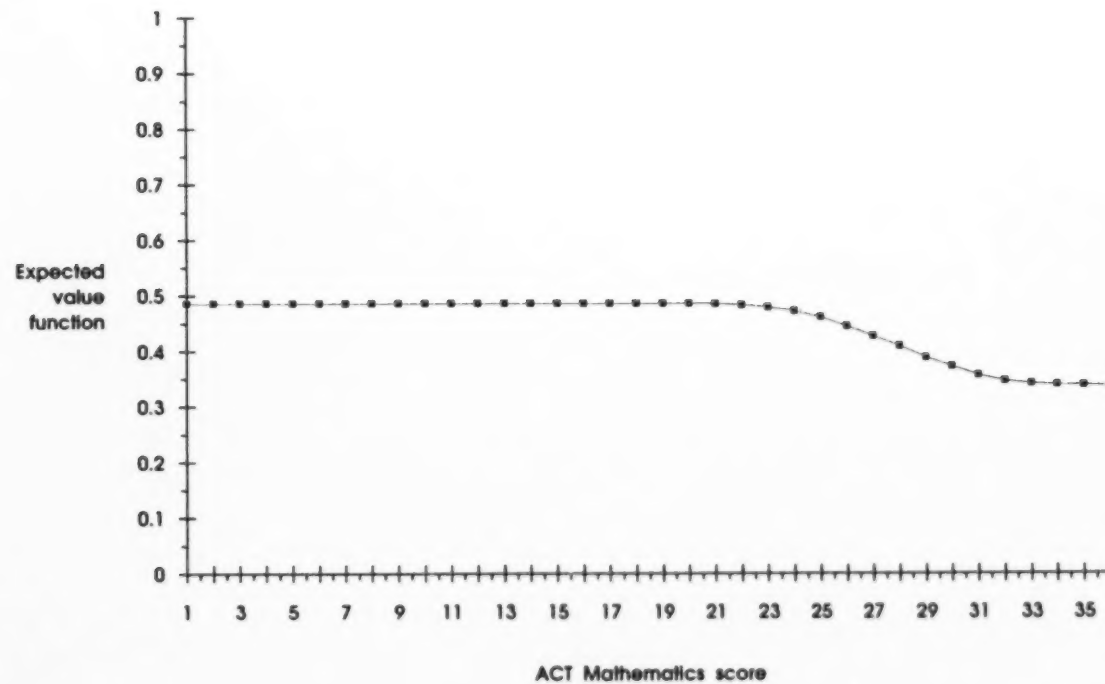


Figure 10.

Conclusions

The current attempt to elicit students' preferences for course grades and remedial course work was somewhat more successful than earlier efforts. The new questionnaire was short enough to be administered in class, thus dramatically improving response rates. Unfortunately, the coherence rate among respondents from the new instrument was lower than from the earlier version. Although the net proportion of usable responses improved, eliciting students' preferences for grades and remedial instruction remains very difficult to do with paper-and-pencil media.

Remedial instruction apparently increased students' chances of success in Course K (Quantitative Methods) and in Course L (Calculus I). Its principal effect was to decrease the proportion of students who received Ws and Fs (by less than 5% each), and to increase the proportion of students who received Ds, Cs, and Bs. In both courses, no increase in the proportion of students earning As was found.

This modest benefit was not large enough, however, to offset the disutilities associated with taking remedial courses, as expressed by students' value functions.

Future research

During the next year, I plan to collect GPA and retention information for the students whose data I have studied. I hope to determine with the new data whether remedial instruction is associated with increased probability of retention or with increased grades in general.

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Appendix A

Statistical Issues

Expected Utility Functions

Consider the class of utility functions defined as a weighted sum of the proportions $p_1, \dots, p_{12}$ defined in Table 1:

$$u(K; \theta) = \sum_{i=1}^{12} w_i p_i(K) \quad (A-1)$$

where $0 \leq w_1, \dots, w_{12} \leq 1$. A utility function cannot be evaluated for the group of students for whom placement decisions are to be made, because the outcomes (future students' test scores and eventual performance in the standard course) are not yet known. In Equation (A-1), for example, the proportions $p_1(K)$, $p_2(K)$, etc., are not known for a particular group of students before they are tested and complete the standard course. These proportions must instead be estimated in some way from data on past students, under the assumption that future students will be similar to past students.

The *expected utility function* is a formal mechanism for dealing with the uncertainty of outcomes in a decision theory model. It is from the expected utility function that decisions on the effectiveness of a placement system can be made. In Bayesian models, an expected utility for a decision d is the average (expected) value of a utility function $u(d, \theta)$ with respect to the decision maker's subjective probability distribution for the outcomes Θ . In the example previously given,

$$E_\theta[u(K, \theta)] = \sum_{i=1}^{12} w_i E_\theta[p_i(K)] \quad (A-2)$$

where $E_\theta[p_i(K)]$, ($i=1, \dots, 12$), are estimated from a past group of students. In the Bayesian model, the expected values of the observed proportions pertain to the decision maker's subjective probability distribution for students' test scores and course grades. In the terminology of Bayesian statistical inference, the subjective joint probability distribution for test scores and course grades of future students is specified by a *predictive density*. The predictive density is based on:

- a. An assumed general probability structure for the joint distribution of θ (test scores and course grades). The joint distribution is described by a *likelihood function* $f(x | \pi)$ involving unknown parameters π of the joint distribution.
- b. Prior beliefs about π , as described by a *prior distribution* $p(\pi)$.
- c. Data x_0 obtained from past students.

A *posterior* distribution $g(\pi \mid x_0)$ for the unknown parameters can be calculated from the likelihood function f , the prior distribution p , and the past data x_0 . Then, by "integrating out" the parameters π , one can calculate a predictive density h :

$$h(x) = \int_{\pi} f(x \mid \pi) g(\pi \mid x_0) d\pi. \quad (\text{A-3})$$

The notation $g(\pi \mid x_0)$ in this equation emphasizes that our subjective beliefs about π are conditioned on the past data x_0 , which is different from the future data x that we have not yet observed. The predictive density, therefore, is based on prior beliefs about the joint distribution of test scores and grades, and on the actual test scores and grades of past students.

Although simple in concept, Bayesian statistical methods can be mathematically formidable in real applications. When prior beliefs are vague or as sample sizes become large, however, Bayesian estimates are, for practical purposes, similar to much simpler estimates based on classical sampling theory (i.e., estimates based only on an assumed model and on data; DeGroot, 1970).

Sawyer (1996) described a simple procedure, based on sampling theory, that can be used to estimate cell probabilities when the outcome in the standard course is dichotomous (i.e., Successful/Unsuccessful). The first step is to estimate the relationship between success in the standard course and a placement test score using a logistic regression function:

$$P[S=1 \mid X=x] = (1 + e^{-\alpha - \beta x})^{-1} \quad (\text{A-4})$$

where $S = 1$, if a student is successful,

$= 0$, if a student is unsuccessful;

and X is the student's score on a placement test or other placement variable. The numbers α and β in Equation (A-4) are unknown parameters that are estimated from data on the test scores and on the success/failure variable Y for a group of enrolled students.

This procedure can be adapted to the model described by Table 1. Let Y denote the final outcome in the standard course:

$Y = -1$, if student does not complete course;

0, if student completes course with grade of F;

1, if student completes course with grade of D;

2, if student completes course with grade of C;

3, if student completes course with grade of B;

4, if student completes course with grade of A.

The numbering of Categories 0-4 of Y was chosen to be consistent with common usage for the grades A -

F. Now, let $T_j = 1$, if $Y \leq j$; and $T_j = 0$, if $Y > j$, ($j=0,1,2,3$) be dichotomous variables. Then,

$$\begin{aligned} P[Y=4 | X=x] &= 1 - P[T_3=1 | X=x] \\ P[Y=j | X=x] &= P[T_j=1 | X=x] - P[T_{j+1}=1 | X=x], \quad (j=0,1,2,3) \\ P[Y=-1 | X=x] &= P[T_{-1}=1 | X=x] \end{aligned} \quad (A-5)$$

Moreover, the conditional probability distributions of T_{-1} , T_0 , T_1 , T_2 , and T_3 , given x can be estimated by logistic regression models like Equation (A-4). The equations in (A-5) provide a way to estimate the conditional probability distribution of Y , given x .

One can use the SAS (1990) LOGISTIC procedure to model $P[T_j=1 | X=x] = 1/(1+\exp(-\alpha_j-\beta x))$ ($j=-1, \dots$), under the assumption that all the dichotomous variables T_j have the same value of β (slope parameter) in their logistic regressions. This procedure is very convenient, as it requires programming only one pass through the data, but the assumption of equal slopes in this application is questionable. To study the feasibility of using the equal slopes method, I estimated $P[Y=j | X=x]$ both ways.

The probabilities $p_1, \dots, p_{12}$ associated with the events in Table 1 can be estimated from the conditional probabilities of success, given test scores, of students who have received remedial instruction and of those who have not. Let

$$\begin{aligned} P_s(y | x) &= P[Y=y | X=x] \text{ for students who enroll} \\ &\quad \text{directly in the standard course,} \\ \text{and let} \quad P_r(y | x) &= P[Y=y | X=x] \text{ for students who enroll} \\ &\quad \text{first in the remedial course.} \end{aligned} \quad (A-6)$$

One hopes that the functions P_S and P_R are quite different; hence, they are estimated separately. From the estimated conditional probabilities $\hat{P}_S(y|x)$ and $\hat{P}_R(y|x)$, expected utilities can be calculated. In the model described by Table 1, the proportion of students associated with Event (12) in Table 1 can be estimated by:

$$\hat{E}_\theta [p_{12}(K)] = \sum_{x \geq K} \hat{P}_S(4|x) \cdot n(x) / N \quad (A-7)$$

where

$\hat{P}_S(4|x)$ = estimated $P[Y = 4 | X = x]$ for students who enrolled directly in the standard course,

K = the minimum score required for enrollment in the standard course (cutoff score),

$n(x)$ = the number of students in the placement group whose test score is equal to x , and

$N = \sum_{x=1}^K n(x)$, the total number of students in the placement group.

The proportions for Events (1) - (11) can be estimated similarly, using the appropriate conditional probability function $\hat{P}_S(y|x)$ or $\hat{P}_R(y|x)$, and the appropriate region $[x \geq K]$ or $[x < K]$.

It is also useful to express the expected utility in another way. Note that

$$\begin{aligned} u(K;\theta) &= \sum_{i=1}^{12} w_i p_i(K) \\ &= (1/N) \cdot \sum_{j=1}^N \sum_{i=1}^{12} w_i I[\theta_j \in E_i], \end{aligned}$$

where N is the number of students in the placement group, $\theta = [\theta_1, \dots, \theta_N]'$, E_i denotes Event (i) in Table 1, and I is an indicator function. Therefore,

$$E_\theta[u(K;\theta)] = (1/N) \cdot \sum_{j=1}^N \sum_{i=1}^{12} w_i P[\theta_j \in E_i]$$

Now, $\theta_j = [X_j, Y_j]'$, where X_j is the placement test score, and Y_j is the outcome (e.g., grade) in the standard course. Therefore,

$$\begin{aligned} E_\theta[u(K;\theta)] &= (1/N) \cdot \sum_{j=1}^N \sum_{i=1}^{12} \sum_{x,y \in E_i} w_i P[X_j=x, Y_j=y] \\ &= (1/N) \cdot \sum_{j=1}^N \sum_{i=1}^{12} \sum_{x,y \in E_i} w_i P[Y_j=y | X_j=x] P[X_j=x] \\ &= (1/N) \sum_{j=1}^N \sum_{y=1}^4 \left\{ \sum_{x < K} w_{y+2} P[Y_j=y | X_j=x] P[X_j=x] + \sum_{x \geq K} w_{y+8} P[Y_j=y | X_j=x] P[X_j=x] \right\} \\ &= (1/N) \sum_{j=1}^N \left\{ \sum_{x < K} E_\theta[vf(1,Y) | X_j=x] P[X_j=x] + \sum_{x \geq K} E_\theta[vf(0,Y) | X_j=x] P[X_j=x] \right\}, \end{aligned}$$

where $E[vf(1,Y) | X=x] = \sum_{y=1}^4 w_{y+2} P_R[Y=y | X=x]$ and $E[vf(0,Y) | X=x] = \sum_{y=1}^4 w_{y+8} P_S[Y=y | X=x]$; and P_R and P_S are as defined in Equation (A-6). Because we can assume that $(X_1, Y_1), \dots, (X_N, Y_N)$ are independent and identically distributed as (X, Y) ,

$$E_\theta[u(K;\theta)] = \sum_{x < K} E_\theta[vf(1,Y) | X=x] P[X=x] + \sum_{x \geq K} E_\theta[vf(0,Y) | X=x] P[X=x].$$

Therefore, an estimate of the expected utility is

$$\hat{E}_\theta[u(K;\theta)] = (1/N) \left\{ \sum_{i: X_i < K} \hat{E}_\theta[vf(1,Y) | X_i=x_i] + \sum_{i: X_i \geq K} \hat{E}_\theta[vf(0,Y) | X_i=x_i] \right\} \quad (A-8)$$

where $\hat{E}_\theta[vf(1,Y) | X_i=x_i] = \sum_{y=1}^4 w_{y+2} \hat{P}_R[Y=y | X=x]$, and $\hat{E}_\theta[vf(0,Y) | X_i=x_i] = \sum_{y=1}^4 w_{y+8} \hat{P}_S[Y=y | X=x]$ are estimated from a past placement group of students.

Effects of Prior Selection

Note that the expected utility function is based on the x -values (e.g., test scores) of all the students in the placement group (the set of students for whom placement decisions are made), not just the students who complete the course. In practical terms, the placement group usually consists of all first-time entering students with test scores, regardless of which course they actually enroll in. Of course, one could also define a placement group for students in a particular program of study (e.g., business) or with particular background characteristics (e.g., minority students).

At an institution with an operational placement system with cutoff score K_0 , we can estimate $P_S(y | x)$ only from data with $x \geq K_0$, and we can estimate $P_R(y | x)$ only from data with $x < K_0$. The reason is that students whose test scores are below the cutoff score K_0 do not enroll directly in the standard course, and therefore do not have performance data unaffected by remedial instruction. Sawyer (1993) noted, however, that the logistic regression model (A-4) can be extrapolated to test scores beyond the range of the data. Schiel & Noble (1993) compared logistic regression functions estimated from truncated subsets of a data set that was not subject to prior selection. They found that when the truncation involved less than 15% of the population, the resulting errors were small, but that large amounts of truncation (e.g., 50%) resulted in large errors. Houston (1993) did computer simulations to examine the effects of

truncation on the accuracy of estimated conditional probabilities of success. He found increases in standard error of 6%, 30%, and 43% when the placement group was truncated at the 25th, 50th, and 75th percentiles, respectively, as compared to the standard error associated with no truncation. In a bootstrapping study, Crouse (1997) found that truncation of 25% or less resulted in fairly modest standard errors in estimating conditional probabilities and accuracy rate, but that truncation of 50% produced large standard errors.

Summary Utility

Whether they rely on single or multiple predictors of students' performance, many postsecondary institutions give students and their counselors final authority in making placement decisions. Therefore, data from operational course placement systems do not typically show a sharp truncation at a cutoff score or cutoff probability, but a trailing off. It would be useful to obtain an estimate of the expected utility of the system not only for alternative cutoff scores or cutoff probabilities, but for the system as it is currently functioning. Note that Equation (A-8) can be restated as

$$SU = (1/N) \left\{ \sum_{[j \in RC]} \hat{E}_\theta[vf(1,Y) | X_j=x_j] + \sum_{[j \in SC]} \hat{E}_\theta[vf(0,Y) | X_j=x_j] \right\} \quad (A-9)$$

where $[j \in RC]$ is the set of students in the remedial course, and $[j \in SC]$ is the set of students in the standard course. Using Equation (A-9), we can obtain an estimate of the expected utility of the placement system with respect to its current actual placement rule, rather than with respect to a cutoff score decision rule.

As before, let x_{min} be the minimum observed test score. The difference $SU - E_\theta[u(x_{min};\theta)]$ is an indicator of whether the placement system as currently operating is of any benefit as a whole, or whether the institution would be better off without the current placement system. In the case of multiple predictors, one could sum over values of p , the probability of success in the standard course, in Equation (A-9).

Appendix B

Value Function Elicitation Questionnaire

Satisfaction With Grades and Course Placement Decisions

To the student:

Having myself taken many courses at the Midwestern University, I know that you are concerned about grades. I am doing some research on your relative satisfaction with different grades in Calculus I. For example, is a B worth almost as much to you as an A, or is it worth only half as much as an A? I also want to find out how satisfied you would be with different grades if you had taken another mathematics course first to help you prepare for this one.

I would very much appreciate your taking a few minutes to answer the questions on the back of this form. Your responses are completely anonymous.

Thank you very much.

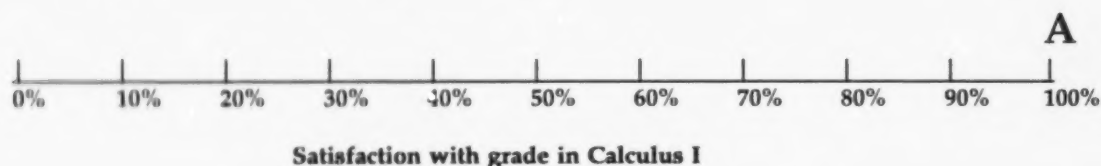
Richard Sawyer,
ACT

Please answer the questions on the back of this form.

1. Think of the end-of-term grade you could get in this course (Calculus I). Use the scale below to show your relative satisfaction with the different possible grades (A, B, C, D, F, or W). The grade of A at the right-hand end of the scale means that you would be 100% satisfied with an A.

First, write the grade of B at an appropriate point on the scale. For example, if you would be about 70% as satisfied with a B as with an A, then write a "B" above the 70% mark. On the other hand, if you would be only half as satisfied with a B as with an A, then write a "B" above the 50% mark. Then, do the same thing for the grades C, D, F, and W. (Note: W stands for "withdrawal"--no credit is given for the course, and the grade is not used in computing your GPA.)

Your responses should reflect only your relative satisfaction with the possible grades in this course. Do not make any assumptions about how the grades are assigned (for example, a grading system where an A represents a certain percentage correct).



2. Have you taken any mathematics courses (including this one) before at the Midwestern University?

_____ YES, I have taken Calculus I before.

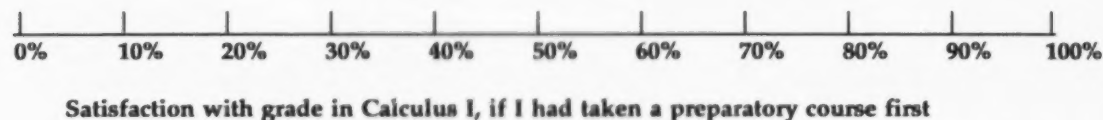
_____ YES, I have taken another mathematics course before at the Midwestern University.

_____ NO. This is the first mathematics course I have taken at the Midwestern University.

If you answered 'YES' to this question, please stop; do not complete the rest of the questionnaire. If you answered 'NO' to this question, please also answer Question 3 below.

3. Suppose you *had* taken another mathematics course at the Midwestern University before enrolling in this course, and that the purpose for taking the other course was to help you earn a higher grade in Calculus I. I want to find out whether your satisfaction with your grade in Calculus I would be affected by your taking the other mathematics course.

As before, think of the grade you could get in Calculus I. Mark the satisfaction scale below with the possible grades A, B, C, D, F, and W, assuming that you had taken a preparatory mathematics course before enrolling in Calculus I. Under this assumption, your satisfaction with each possible grade might be less than what you indicated in Question 1, because taking a preparatory course requires additional time, money, and effort on your part.



Thank you for your help!

Appendix C

Logistic Regression Results

Table C-1.
Logistic Regression of Success Variables on ACT Mathematics Score,
by Course and Prior Remedial Instruction

Course	Group	N	B or Higher		C or Higher	
			Intercept (std. error)	Regression wgt. (std. error)	Intercept (std. error)	Regression wgt. (std. error)
Course I: Elementary Functions	With prior rem. instr.	17	-3.737 (3.966)	0.092 (0.201)	-0.396 (2.059)	0.002 (0.110)
	No prior rem. instr.	561	-3.171 (0.644)	0.108 (0.027)	-1.638 (0.560)	0.087 (0.024)
Course K: Quantitative Methods	With prior rem. instr.	343	-4.755 (0.923)	0.182 (0.043)	-1.768 (0.723)	0.110 (0.035)
	No prior rem. instr.	1943	-5.168 (0.374)	0.204 (0.016)	-3.202 (0.370)	0.177 (0.016)
Course L: Calculus I	With prior rem. instr.	126	-0.977 (1.184)	0.008 (0.051)	-1.950 (1.158)	0.102 (0.050)
	No prior rem. instr.	1082	-5.755 (0.568)	0.211 (0.022)	-3.267 (0.524)	0.156 (0.021)

Table C-2.
Logistic Regression of Variables T_{-1} , ..., T_3 on ACT Mathematics Score,
by Course and Prior Remedial Instruction

(Equal Slopes Model)

Course	Group	N	Intercepts (std. errors)					Slope (std. error)
			T_{-1}	T_0	T_1	T_2	T_3	
Course I: Elementary Functions	With prior rem. instr.	17	-1.10 (1.83)	-0.53 (1.81)	0.20 (1.80)	1.26 (1.83)	2.85 (2.02)	-0.004 (0.096)
	No prior rem. instr.	561	0.29 (0.48)	0.89 (0.48)	1.54 (0.48)	2.58 (0.49)	3.85 (0.50)	-0.093 (0.021)
Course K: Quantitative Methods	With prior rem. instr.	342	0.89 (0.62)	1.30 (0.61)	2.21 (0.62)	3.65 (0.64)	5.17 (0.67)	-0.145 (0.030)
	No prior rem. instr.	1938	2.36 (0.29)	2.89 (0.29)	3.59 (0.29)	4.96 (0.30)	6.35 (0.32)	-0.205 (0.013)
Course L: Calculus I	With prior rem. instr.	126	0.34 (0.98)	0.86 (0.98)	1.39 (0.98)	2.52 (1.00)	4.53 (1.04)	-0.088 (0.042)
	No prior rem. instr.	1082	2.78 (0.43)	3.37 (0.43)	3.77 (0.43)	4.82 (0.44)	6.10 (0.46)	-0.190 (0.017)

Table C-3.
Logistic Regression of Variables T_{-1} , ..., T_3 on ACT Mathematics Score,
by Course and Prior Remedial Instruction

(Separate Slopes Model)

Course	Group	N	T_{-1}		T_0		T_1	
			Intrcpt. (std. error)	Reg. wgt. (std. error)	Intrcpt. (std. error)	Reg. wgt. (std. error)	Intrcpt. (std. error)	Reg. wgt. (std. error)
Course I: Elementary Functions	With prior rem. instr.	17	-1.04 (2.36)	-0.01 (0.13)	1.60 (2.10)	-0.12 (0.12)	1.00 (2.08)	-0.05 (0.11)
	No prior rem. instr.	561	-0.52 (0.71)	-0.06 (0.03)	-0.16 (0.61)	-0.05 (0.03)	1.37 (0.56)	-0.08 (0.02)
Course K: Quantitative Methods	With prior rem. instr.	342	-0.78 (0.98)	-0.06 (0.05)	-0.19 (0.85)	-0.07 (0.04)	1.35 (0.73)	-0.10 (0.04)
	No prior rem. instr.	1938	2.23 (0.55)	-0.20 (0.02)	2.45 (0.46)	-0.19 (0.02)	2.78 (0.38)	-0.17 (0.02)
Course L: Calculus I	With prior rem. instr.	126	1.09 (1.35)	-0.12 (0.06)	1.57 (1.24)	-0.12 (0.05)	2.16 (1.19)	-0.12 (0.05)
	No prior rem. instr.	1082	0.78 (0.65)	-0.10 (0.03)	1.78 (0.56)	-0.12 (0.02)	3.05 (0.53)	-0.15 (0.02)

(continued on next page)

Table C-3 (cont'd).
Logistic Regression of Variables T_1 , ..., T_3 on ACT Mathematics Score,
by Course and Prior Remedial Instruction

(Separate Slopes Model)

Course	Group	N	T_2		T_3	
			Intrcpt. (std. error)	Reg. wgt. (std. error)	Intrcpt. (std. error)	Reg. wgt. (std. error)
Course I: Elementary Functions	With prior rem. instr.	17	-1.90 (2.21)	0.18 (0.13)	5.24 (6.26)	-0.13 (0.31)
	No prior rem. instr.	561	3.05 (0.62)	-0.11 (0.03)	5.32 (0.93)	-0.15 (0.04)
Course K: Quantitative Methods	With prior rem. instr.	342	4.63 (0.88)	-0.19 (0.04)	7.23 (1.39)	-0.24 (0.06)
	No prior rem. instr.	1938	4.96 (0.37)	-0.20 (0.02)	7.65 (0.50)	-0.26 (0.02)
Course L: Calculus I	With prior rem. instr.	126	1.09 (1.14)	-0.03 (0.05)	3.30 (2.00)	-0.04 (0.08)
	No prior rem. instr.	1082	5.50 (0.56)	-0.21 (0.02)	8.46 (0.75)	-0.27 (0.03)